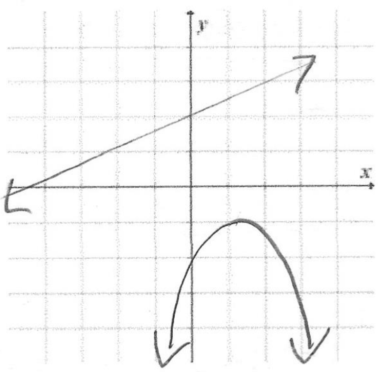
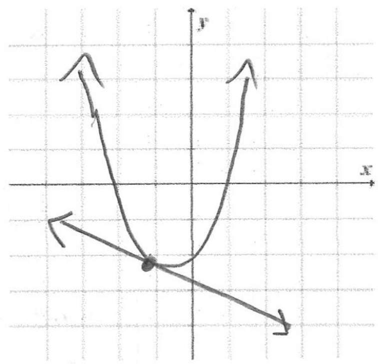
Systems of Equations

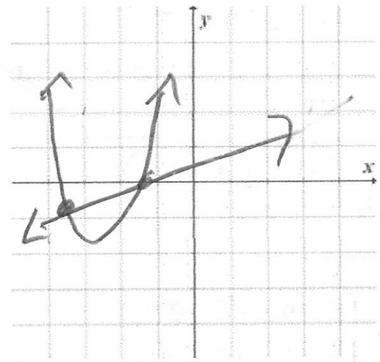
7.3 Notes: Nonlinear Systems MAT133_Lawson

Many of the following examples come from College Algebra Handbook, College of the Ozarks,2019

A **non-linear systems of equations** is one in which at least one equation of the system is not a linear equation. In this section we will consider two-by-two systems containing equations whose graphs result in figures such as lines, parabolas, circles, etc. It is often helpful to think about the graphs of the equations before solving the system. This may help you determine the number of solutions to expect. For example, if you are solving a system of equations involving a linear equation and a quadratic equation, the two curves might not intersect, they might intersect in exactly one point, or they might intersect in two points. Roughly sketch each of those possibilities below.







(Example 1) Find all solutions to:

(a) $\begin{cases} y = x^2 - x - 1 \\ 3x - y = 4 \rightarrow y = 3x - 4 \end{cases}$

$3x - 4 = x^2 - x - 1$

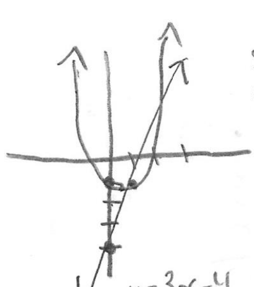
$0 = x^2 - 4x + 3$

$0 = (x-1)(x-3)$

$x = 1 \rightarrow y = 3(1) - 4 = -1$

$x = 3 \rightarrow x = 3(3) - 4 = 5$

$(1,-1)$
$(3,5)$



$y = x^2 - x - 1$

$y = (x^2 - x + \frac{1}{4}) - 1 - \frac{1}{4}$

$(x - \frac{1}{2})^2 - \frac{1}{4}$

(b) $\begin{cases} (x+3)^2 + (y-4)^2 = 20 \rightarrow x^2 + 6x + 9 + y^2 - 8y + 16 = 20 \\ (x+4)^2 + (y-3)^2 = 26 \rightarrow x^2 + 8x + 16 + y^2 - 6y + 9 = 26 \end{cases}$

$x^2 + y^2 + 6x - 8y + 5 = 0$

$-(x^2 + y^2 + 8x - 6y - 1 = 0)$

$-2x - 2y + 6 = 0$

$-2y = -6 + 2x$

$y = 3 - x$

$\rightarrow x^2 + (3-x)^2 + 6x - 8(3-x) + 5 = 0$

$x^2 + 9 - 6x + x^2 + 6x - 24 + 8x + 5 = 0$

$2x^2 + 8x - 10 = 0$

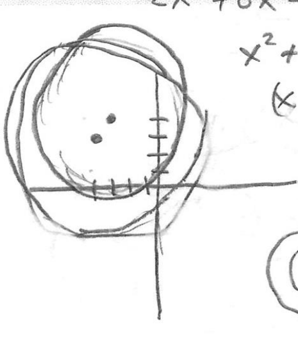
$x^2 + 4x - 5 = 0$

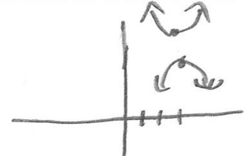
$(x-1)(x+5) = 0$

$x = 1 \rightarrow y = 2$

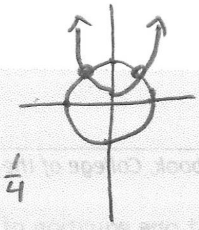
$x = -5 \rightarrow y = 8$

$(1,2), (-5,8)$





(Example 2) Find all solutions to:



(c) $\begin{cases} y^2 + x^2 &= 4 \\ 4y - x^2 &= 1 \end{cases} \rightarrow y = \frac{1}{4}x^2 + \frac{1}{4}$

$4y - 1 = x^2$

$y^2 + 4y - 1 = 4$

$y^2 + 4y - 5 = 0$

$(y-1)(y+5) = 0$

$y = 1 \rightarrow x^2 = 3 \rightarrow x = \pm\sqrt{3}$

$y = -5 \rightarrow x^2 = -21 \rightarrow x = \cancel{\pm\sqrt{-21}}$ not real

$\boxed{(\sqrt{3}, 1), (-\sqrt{3}, 1)}$

(d) $\begin{cases} y + x^2 = 6x \\ y - 11 = (x-3)^2 \end{cases}$

$y = -x^2 + 6x = -(x^2 - 6x + 9) + 9$

$= -(x-3)^2 + 9$

$y = (x-3)^2 + 11$

$-x^2 + 6x - 11 = x^2 - 6x + 9$

$x^2 - 6x + 10 = 0$

$x = \dfrac{6 \pm \sqrt{36-40}}{2} = \dfrac{6 \pm \sqrt{4}}{2} = 3 \pm i$

$\boxed{\text{no real solutions}}$

(e) $\begin{cases} y - 9 = e^{2x} \rightarrow y = e^{2x} + 9 \\ 3 = y - 7e^x \rightarrow y = 7e^x + 3 \end{cases}$

$e^{2x} + 9 = 7e^x + 3$

$e^{2x} - 7e^x + 6 = 0$ ("like" a quadratic)

Let $u = e^x$

Then,

$\rightarrow u^2 - 7u + 6 = 0$

$(u-6)(u-1) = 0$

$u = 6 \rightarrow e^x = 6 \rightarrow x = \ln 6$

$u = 1 \rightarrow e^x = 1 \rightarrow x = \ln 1 = 0$

$x = \ln 6 \rightarrow y = 7e^{\ln 6} + 3 = 7 \cdot 6 + 3 = 45$

$x = 0 \rightarrow y = 7e^0 + 3 = 7 + 3 = 10$

$(\ln 6, 45)$
$(0, 10)$

(f) $\begin{cases} y = \log(x+4) + 1 \\ y - 2 = -\log(x+7) \rightarrow y = 2 - \log(x+7) \end{cases}$

$\log(x+4) + 1 = 2 - \log(x+7)$

$\log(x+4) + \log(x+7) = 1$

$\log(x+4)(x+7) = 1$

$(x+4)(x+7) = 10$

$x^2 + 11x + 28 = 10$

$x^2 + 11x + 18 = 0$

$(x+9)(x+2) = 0$

$\cancel{x = -9 \rightarrow y = \log(-5) + 1}$

$x = -2 \rightarrow y = \log(2) + 1$

$(-2, \log(2) + 1)$

or

$(-2, 2 - \log(5))$

$\approx (-2, 1.3)$